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Design of a Tubular Heat Exchanger

In section 4.1, we examined a variety of heat exchange equipment used in the food process industry. There are a number of different geometrical configurations used in designing heat exchange equipment, such as tubular, plate, and scraped surface heat exchangers. The primary objective in using a heat exchanger is to transfer thermal energy from one fluid to another. Recall from previous discussion that the change in heat energy in a fluid, if its temperature changes from T_1 to T_2, may be expressed as;

(1)

Where m = mass flow rate of a fluid, kg/s, c_p = specific heat of a fluid, kJ/kg°C and the temperature change of a fluid from some initial temperature T_1 to a final temperature T_2.

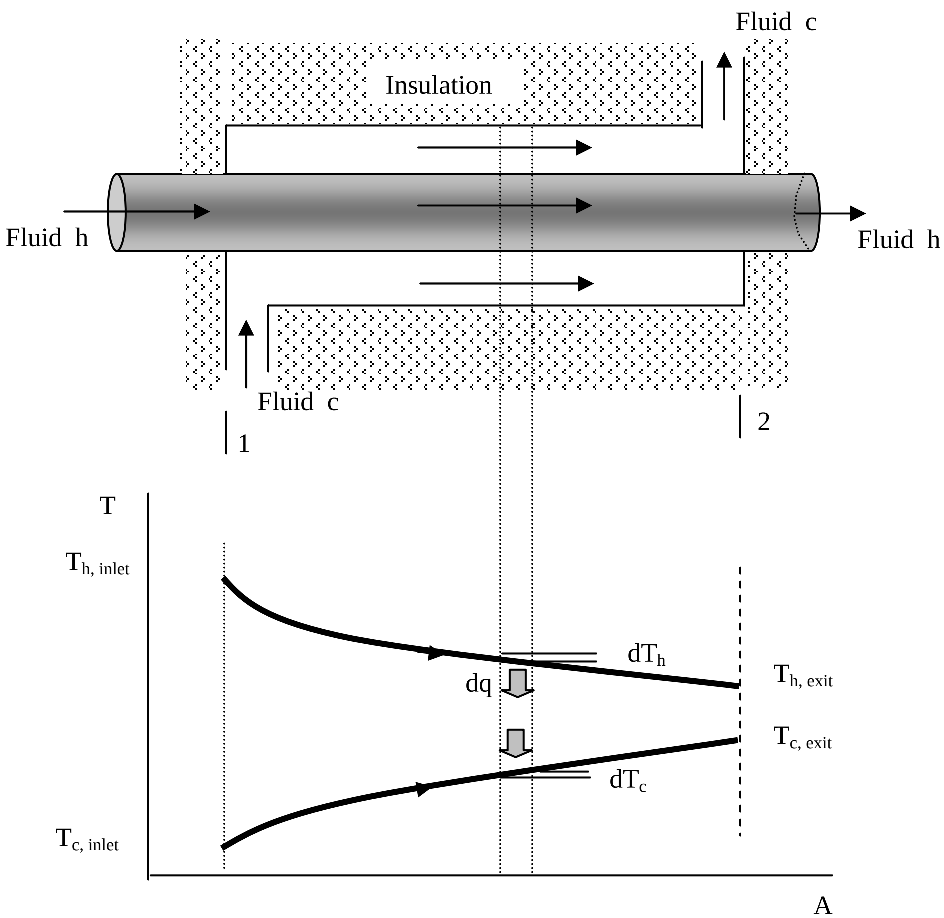


Figure 1. A co-current (or parallel) heat exchanger

Let us consider a tubular heat exchanger as shown in Figure 1. A fluid "h" enters the heat exchanger at location "1" and flows through the inner pipe and exits at location "2". Its temperature changes from $T_{h,\,inlet}$ to $T_{h,exit}$. The second fluid "c" enters the annular space between the outer and the inner pipe of the tubular heat exchanger at location 1 and exits at location 2. Its temperature changes from T_{c1} to T_{c2}. The outside of the heat exchanger is insulated. Because the heat transfer occurs only between fluids "h" and "c", the decrease in the heat energy of fluid "h" must equal the increase in that of fluid "c". Therefore, from an energy balance, the rate of heat transfer between the fluids is:

$$q = \dot{m}_h c_{ph} (T_{h,inlet} - T_{h,exit}) = \dot{m}_c c_{pc} (T_{c,exit} - T_{c,inlet}) \quad (2)$$

Equation 2 is useful if we are interested in determining the inlet and exit temperatures of the two fluid streams. Furthermore, we may use this equation to determine the mass flow rate of either fluid stream, provided all other conditions are known. But, this equation does not provide us with any information about the size of the heat exchanger required for accomplishing a desired rate of heat transfer. And, we cannot use it to determine the resistance to heat transfer between the two fluid streams. For those cases, we need to determine heat transfer perpendicular to the flow of the fluid streams.

Consider a thin slice of the heat exchanger as shown in Figure 1. We want to determine the rate of heat transfer from fluid "h" to "c", perpendicular to the direction of the fluid streams. For this thin slice of the heat exchanger, the rate of heat transfer, dq, from fluid "h" to fluid "c" may be expressed as:

(3)

Where ΔT is the temperature difference between fluid "h" and fluid "c". Note that this temperature difference, ΔT, varies from side 1 to side 2 of the heat exchanger. At the inlet of the fluid stream "h", the temperature gradient ΔT is $T_{h,inlet}$-$T_{c,inlet}$ and on side 2, it is $T_{h,exit}$ - $T_{c,exit}$ (see Figure 1). To solve Eqn (3) we need some average value of ΔT that represents the temperature gradient, perpendicular to the direction of flow. Although it may be tempting to take an arithmetic average of the two ΔT values, at locations (1) and (2), the arithmetic average value will be incorrect, because as seen in the Fig (1), the temperature plots are non linear. Therefore, we develop the following mathematical analysis to determine a value of ΔT that will correctly describe the "average" temperature difference between the fluids "h" and "c" as they flow through the heat exchanger.

Since the temperature gradient ΔT is

(4)

we can write Eqn (4) in a differential form as

$$d(\Delta T) = dT_h - dT_c \quad (5)$$

From Eqn (2), for the small slice of heat exchanger, for fluid stream "h",

(6)

and, for fluid stream "c"

$$dq = \dot{m}_c c_{pc} dT_c \quad (7)$$

substituting Eqns (6) and (7) in Eqn (5):

$$d(\Delta T) = -dq\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right) \quad (8)$$

Substituting Eqn (3) in Eqn (8),

$$d(\Delta T) = -U\Delta T\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)dA \quad (9)$$

Then separating variables in Eqn (9),

$$\frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)dA \quad (10)$$

Integrating, between the limits as area of heat exchange increases from 0 to A, the temperature gradient changes from ΔT_1 to ΔT_2

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)\int_0^A dA \quad (11)$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -U\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)A \quad (12)$$

Substituting Eqn (2) in Eqn (12)

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{T_{h,inlet} - T_{h,exit}}{q} + \frac{T_{c,exit} - T_{c,inlet}}{q}\right) \quad (13)$$

Simplifying,

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -\frac{UA}{q}\left((T_{h,inlet} - T_{c,inlet}) - (T_{h,exit} - T_{c,exit})\right) \quad (14)$$

Since
$$\begin{aligned} T_{h,inlet} - T_{c,inlet} &= \Delta T_1 \\ T_{h,exit} - T_{c,exit} &= \Delta T_2 \end{aligned} \quad (15)$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -\frac{UA}{q}(\Delta T_1 - \Delta T_2) \quad (16)$$

(17)

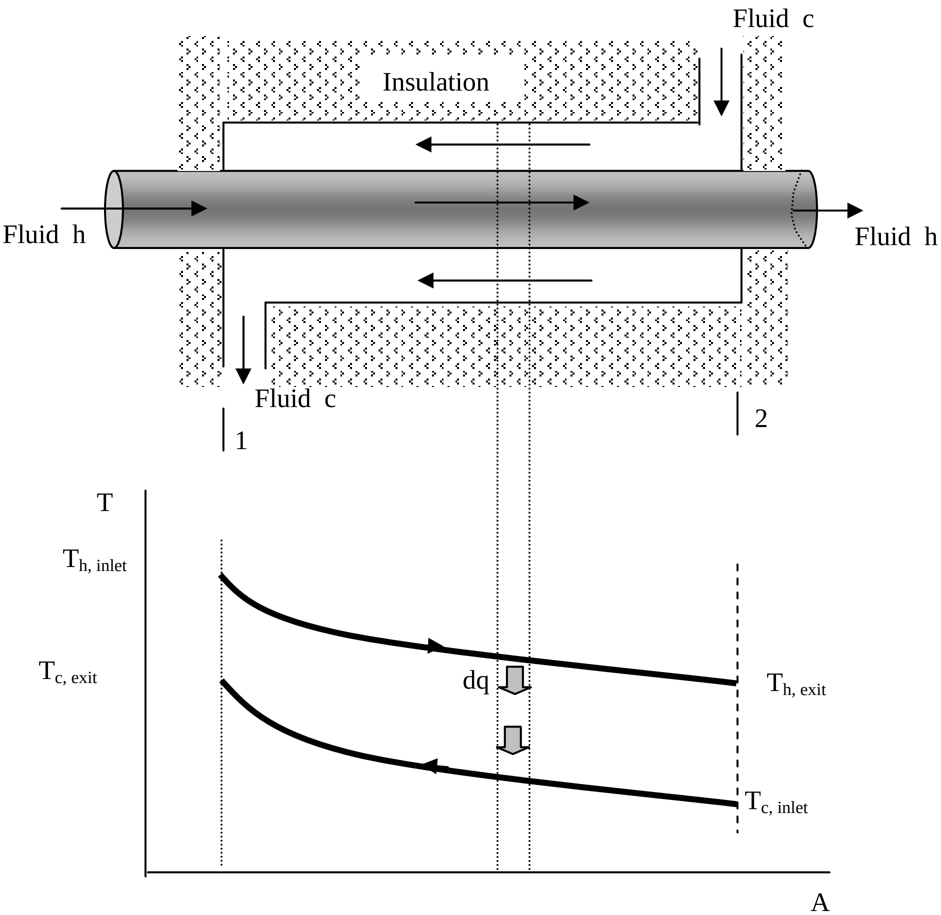
$$\boxed{q = UA(\Delta T_{lm})} \quad (18)$$

where

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} \quad (19)$$

Equation 18 may be used to design a heat exchanger, determine its area, and the overall resistance to heat transfer.

A Counter-Current Heat Exchanger



Class Problem

Calculate the log mean temperature difference for a heat exchanger with the following data, the temperature of hot stream decreases from 150°C to 50°C, while the temperature of cold stream increases from 40°C to 80°C.

Consider another case when temperature of hot stream decreases from 150°C to 50°C and cold stream temperature increases from 40°C to 45°C.