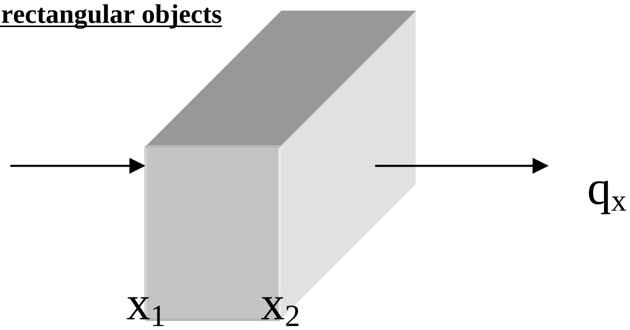
STEADY -STATE HEAT CONDUCTION

Steady state means that temperature in an object may vary by location

Conduction in rectangular objects



Fourier's law describes the rate of heat conduction in a solid.

Boundary Conditions

$x = x_1 \quad T = T_1$
$x = x_2 \quad T = T_2$

Separating the variables,

Integrating from x_1 to x (some interior location within the slab)

$$\int_{x_1}^{x} \frac{q_x}{A} dx = -\int_{T_1}^{T} k dT$$

$$T = T_1 - \frac{q_x}{kA}(x - x_1)$$

$$q_x = -kA\frac{(T - T_1)}{(x - x_1)}$$

if integrated from x_1 to x_2

where L = thickness of the slab.

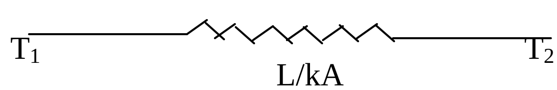
Thermal Resistance concept:

In the flow of electricity, the electrical resistance is determined from the ratio of electric potential divided by the electric current. Similarly, we may consider heat resistance as a ratio of the driving potential (temperature difference) divided by the rate of heat transfer.

Thus,

$$R_{conduction} = \frac{T_1 - T_2}{q_x}$$

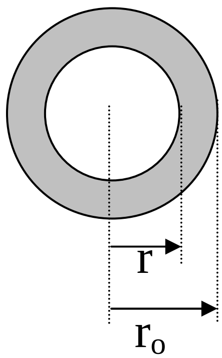
or,

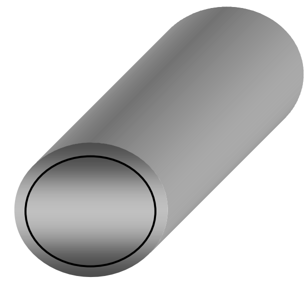


units of resistance = °C/W

Conduction in Cylindrical objects

HEAT TRANSFER IN A TUBULAR PIPE





Fourier's Law in cylindrical coordinates

$$q_r = -kA\frac{dT}{dr}$$

Boundary Conditions

$T = T_i$ at $r = r_i$

$T = T_o$ at $r = r_0$

Separating the variables

Integrating

$$\frac{q}{2\pi L}\left|\ln r\right|_{r_i}^{r_o} = -k\left|T\right|_{T_i}^{T_o}$$

$$\frac{q}{2\pi L}\left|\ln r_0 - \ln r_i\right| = -k\left(T_0 - T_i\right)$$

$$q = \frac{2\pi Lk(T_i - T_o)}{\ln\left(\frac{r_o}{r_i}\right)}$$

Using the preceding equation, we may write the conductive resistance in cylindrical coordinates as: